

# Future singularity avoidance in phantom dark energy models

Jaume de Haro<sup>a,\*</sup>

3rd March 2013

<sup>a</sup>Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, Diagonal 647, 08028 Barcelona, Spain

## Abstract

Different approaches to quantum cosmology are studied in order to deal with the future singularity avoidance problem. Our results show that these future singularities will persist but could take different forms. As an example we have studied the big rip which appear when one considers the state equation  $P = \omega\rho$  with  $\omega < -1$ , showing that it does not disappear in modified gravity. On the other hand, it is well-known that quantum geometric effects (holonomy corrections) in loop quantum cosmology introduce a quadratic modification, namely proportional to  $\rho^2$ , in Friedmann's equation that replace the big rip by a non-singular bounce. However this modified Friedmann equation could have been obtained in an inconsistent way, what means that the obtained results from this equation, in particular singularity avoidance, would be incorrect. In fact, we will show that instead of a non-singular bounce, the big rip singularity would be replaced, in loop quantum cosmology, by other kind of singularity.

**Pacs numbers:** 98.80.Qc, 04.20.Dw, 04.62.+v

*1.- Introduction*— Studies of distant type Ia supernovae [1, 2] indicates that the dominant part of the energy of the universe must be gravitationally repulsive driving our universe expanding in an accelerating way. To explain this acceleration one usually assumes the existence of dark energy with a negative pressure, in general one can assume a perfect fluid with state equation  $P = \omega\rho$ , with  $\omega < -1/3$  in order to have cosmic acceleration. Moreover, observations from WMAP indicates the value  $\omega \cong -1.10$  [3], what means that our universe would be dominated by “phantom energy” ( $\omega < -1$ ). However, the classical solutions of general relativity for a Friedmann-Robertson-Walker (FRW) model containing dark energy lead, in general, to future singularities [4, 5, 6] (big rip, future sudden singularities, etc.). Lately, a good number of papers have been dealing with the possibility of

---

\*E-mail: jaime.haro@upc.edu

avoiding these future singularities, using different approaches to quantum cosmology like loop quantum cosmology, semiclassical gravity, modified gravity, brane cosmology, etc. This paper has two main objectives: The first one is to discuss this different approaches, to show in which way they modify the dynamics of our universe and to check if, effectively, they could avoid the future singularities that appear in classical cosmology. And the second one is to show that the modified Friedmann equation in loop quantum cosmology could have been obtained in an inconsistent way. And thus, the current statement that, in loop quantum cosmology, the big rip singularity is replaced by a non-singular bounce would be incorrect.

The units used in this paper are  $\hbar = c = M_p = 1$  being  $M_p$  the reduced Planck mass.

*2.- Einstein cosmology*— For the flat FRW spacetime filled by a perfect fluid with state equation  $P = f(\rho)$ , Einstein theory, is obtained from the Lagrangian  $\mathcal{L} = \frac{1}{2}Ra^3 - \rho a^3$  where  $R = 6(\dot{H} + 2H^2)$  is the scalar curvature,  $a$  is the scale factor and  $H = \frac{\dot{a}}{a}$  is the Hubble parameter.

This Lagrangian has been constructed in co-moving fluid coordinates (see [7] and Section III C of [8]), and the energy density  $\rho$  has to be understood as a function of the scalar factor  $a$ . This relation comes from the consevation equation

$$d(\rho a^3) = -Pd(a^3) \iff \dot{\rho} = -3H(\rho + P) \iff \frac{d\rho}{da} = -\frac{3}{a}(\rho + P). \quad (1)$$

Since the state equation that we are studying is  $P = f(\rho)$  one has the differential equation

$$\frac{d\rho}{\rho + f(\rho)} = -\frac{3}{a}da, \quad (2)$$

that after integration gives  $\rho$  as a function of  $a$ . For example, when  $P = \omega\rho$  one has

$$\rho(a) = \rho_0 \left( \frac{a}{a_0} \right)^{-3(1+\omega)}, \quad (3)$$

where  $\rho_0$  is the value of  $\rho$  when  $a = a_0$ .

The Lagrangian can be written as follows  $\mathcal{L} = 3 \left( \frac{d(\dot{a}a^2)}{dt} - \dot{a}^2 a \right) - \rho a^3$ , this means that the same theory is obtained avoiding the total derivative, which gives the Lagrangian  $\mathcal{L}_E = -3H^2 a^3 - \rho a^3$ . The conjugate momentum is then given by  $p = \frac{\partial \mathcal{L}_E}{\partial \dot{a}} = -6Ha^2$ , and thus the Hamiltonian is

$$\mathcal{H}_E = \dot{a}p - \mathcal{L}_E = -3H^2 a^3 + \rho a^3. \quad (4)$$

In general relativity the Hamiltonian is constrained to be zero, which gives the Friedmann equation

$$H^2 = \rho/3, \quad (5)$$

that together with the conservation equation are the dynamical equations that describe the evolution of the universe.

The Raychaudury equation is obtained from the Hamilton equation  $\dot{p} = -\frac{\partial \mathcal{H}_E}{\partial a}$ , which gives

$$\dot{p} = -\frac{p^2}{12a^2} - \frac{\partial(\rho a^3)}{\partial a} = -\frac{p^2}{12a^2} + 3Pa^2, \quad (6)$$

where we have used the conservation equation. Then from the Friedmann equation one easily obtains  $\dot{H} = -\frac{1}{2}(\rho + P)$ .

A solvable example is the case of a barotropic perfect fluid with state equation  $P = \omega\rho$  [4], which gives

$$H(t) = \frac{2}{3(1+\omega)} \frac{1}{t-t_s} \quad \rho(t) = \frac{4}{3(1+\omega)^2} \frac{1}{(t-t_s)^2}, \quad (7)$$

where  $t_s \equiv t_0 - \frac{2}{3H_0(1+\omega)}$ , being  $H_0 = H(t_0)$  the initial condition. Then, if one assumes  $H_0 > 0$  and  $\omega < -1$  one has  $t_s > t_0$ , and thus, one has a big rip singularity.

**3.- Modified gravity**— An alternative to Einstein cosmology is modified gravity, where higher-curvature terms are taken into account. The theory is based in the Lagrangian (see for instance [9, 10, 11])  $\mathcal{L}_{MG} = f(R)a^3 - a^3\rho$ , and to find the Hamiltonian formulation, one can use the Ostrogradskii's construction [12] introducing the variables  $a_1 \equiv a$  and  $a_2 \equiv \dot{a}$ , and the momenta

$$p_1 \equiv \frac{\partial \mathcal{L}_{MG}}{\partial \dot{a}} - \frac{d}{dt} \frac{\partial \mathcal{L}_{MG}}{\partial \ddot{a}} = -6a^2 f''(R) \dot{R}, \quad p_2 \equiv \frac{\partial \mathcal{L}_{MG}}{\partial \ddot{a}} = 6a^2 f'(R). \quad (8)$$

Then, the Hamiltonian in modified gravity is given by

$$\mathcal{H}_{MG} \equiv p_1 \dot{a} + p_2 \ddot{a} - \mathcal{L}_{MG} = \left( -6f''(R) \dot{R} H + f'(R)(R - 6H^2) - f(R) + \rho \right) a^3, \quad (9)$$

and the Hamiltonian constraint  $\mathcal{H}_{MG} = 0$  gives the modified Friedmann equation

$$-6f''(R) \dot{R} H + f'(R)(R - 6H^2) - f(R) + \rho = 0, \quad (10)$$

that with the conservation equation  $\dot{\rho} = -3H(P + \rho)$  gives the dynamics of the universe in modified gravity.

Note that, the Hamilton equations  $\dot{a}_1 = \frac{\partial \mathcal{H}_{MG}}{\partial p_1}$ ,  $\dot{a}_2 = \frac{\partial \mathcal{H}_{MG}}{\partial p_2}$  and  $\dot{p}_2 = -\frac{\partial \mathcal{H}_{MG}}{\partial a_2}$  are identities. The dynamical equation, i.e. the Euler-Lagrange equation, is  $\dot{p}_1 = -\frac{\partial \mathcal{H}_{MG}}{\partial a_1}$  which gives the modified Raychaudury equation.

**Remark 0.1.** A more general theory consist in modified Gauss-Bonnet gravity which is based on the Lagrangian  $\mathcal{L}_{GB} = f(R, G)a^3 - a^3\rho$ , being  $G = 24H^2(\dot{H} + H^2)$  the scalar Gauss-Bonnet curvature (see for instance [13]).

As special case we can consider  $f(R) = \frac{R}{2} - \frac{\alpha}{12}R^2$ . Then, equation (10) becomes

$$H^2 = \frac{\rho}{3} + 2\alpha(3H^2\dot{H} + H\ddot{H} - \frac{1}{2}\dot{H}^2), \quad (11)$$

which coincides with the semiclassical Friedmann equation, obtained taking into account quantum effects due to a massless conformally coupled field when one chooses the other parameter, namely  $\beta$ , equal to zero (see for instance [14]).

For the state equation  $P = \omega\rho$ , the case  $\omega < -1, \alpha > 0$  was studied in great detail in [14]. There, the main obtained result is that almost all the solution have future singularities in the contracting phase, having the following behavior:

$$H(t) \sim -\frac{1}{2(t_s - t)}, \quad \rho(t) \sim 0, \quad \text{for } t < t_s \quad (12)$$

being  $t_s$  the time at which the future singularity appears. This behavior means that if the universe is initially in the expanding phase, like nowadays, it will bounce and will enter in the contracting phase where it will develop the singularity described by (12).

On the other hand, the case  $\omega < -1, \alpha < 0$  is completely different, because now the expanding and contracting phase decouple, that is, the bounces are not allowed. To show this, we consider the new variable  $\bar{p}^2 = \epsilon H$  where  $\epsilon = \text{sign}(H)$ . Using this variable the modified Friedmann equation becomes

$$\begin{aligned} \frac{d}{dt} (\bar{p}^2/2 + V(\bar{p}, \rho)) &= -3\epsilon\bar{p}^2\dot{\bar{p}}^2 + \frac{\epsilon}{8\alpha}(1 + \omega)\rho \\ \iff \ddot{\bar{p}} &= -\partial_{\bar{p}}V(\bar{p}, \rho) - 3\epsilon\bar{p}^2\dot{\bar{p}}, \end{aligned} \quad (13)$$

where  $V(\bar{p}, \rho) = -\frac{1}{8\alpha}(\bar{p}^2 + \frac{\rho}{3\bar{p}^2})$ . Then, since  $V(0, \rho) = +\infty$  this means that the universe cannot bounce from one phase to another.

Once we have proved that the universe cannot bounce, the next step is to look for singular solution in the expanding phase and compare them with the classical ones (equation (7)). To do this, we look for solutions with the following behavior at late times  $\rho(t) \sim \rho_0(t_s - t)^{-\nu}$  with  $\nu > 0$ . Inserting this solution in the conservation equation one obtains  $H(t) \sim \frac{\nu}{3(1+\omega)(t_s - t)}$ , and finally, inserting both expressions in the modified Friedmann equation (equation (11)) and retaining the leading terms, one gets  $\nu = -4$  and  $\rho_0 = -\frac{18\alpha(3\omega-5)}{3(\omega+1)^3} > 0$ . Thus,

$$H(t) \sim -\frac{4}{3(\omega+1)(t_s - t)}, \quad \rho(t) \sim \frac{\rho_0}{(t_s - t)^4}. \quad (14)$$

Comparing this solution with (7) we deduce that, in that case, modified gravity make worse the singularities.

To end this Section, note that for more complicated functions  $f(R)$ , it seems impossible to perform a qualitative analysis of the dynamics. In such cases only numerical simulations could show the behavior of our universe at late times.

4.- *Loop quantum cosmology*—An approach to quantum cosmology that could avoid the big rip singularity is loop quantum cosmology, where for states which correspond to a macroscopic universe, such as ours, at late times the following effective Hamiltonian, which captures the underlying loop quantum dynamics, is considered [15, 16, 17]

$$\mathcal{H}_{LQC} = -3V \frac{\sin^2(\lambda\beta)}{\gamma^2\lambda^2} + V\rho, \quad (15)$$

where  $\gamma \cong 0.2375$  is the Barbero-Immirzi parameter [18] and  $\lambda$  is a parameter with dimensions of length, which is determined invoking the quantum nature of the geometry, that is, identifying its square with the minimum eigenvalue of the area operator in LQG, which gives as a result  $\lambda \equiv \sqrt{\frac{\sqrt{3}}{4}}\gamma$  (see [17]). In (15),  $V$  is the physical volume  $V = a^3$  and  $\beta$  is canonically conjugate to  $V$  and satisfies  $\{\beta, V\} = \frac{\gamma}{2}$ , where  $\{\cdot, \cdot\}$  is the Poisson bracket, which for the canonically conjugate variables  $(a, p = -6Ha^2)$  takes the form  $\{\beta, V\} = \frac{\partial\beta}{\partial a} \frac{\partial V}{\partial p} - \frac{\partial\beta}{\partial p} \frac{\partial V}{\partial a} = -3a^2 \frac{\partial\beta}{\partial p}$ . Then, since  $\{\beta, V\} = \frac{\gamma}{2}$  one can conclude that  $\beta = \gamma H$ .

**Remark 0.2.** *This last statement does not seems clear in some papers of loop quantum cosmology. For example, in [19] in order to define  $\beta$  the authors assert that “On the classical solution  $\beta$  is related to the scalar factor as  $\beta = \gamma H$ ” and in [20] it is stated that “The variable  $\beta$ , in the limit  $\beta \rightarrow 0$ , is linked to the Hubble factor via the relation  $\beta = \gamma H$ ”. We will discuss later what really happens with the relation between  $\beta$  and  $H$ .*

The Hamiltonian constraint is then given by  $\frac{\sin^2(\lambda\beta)}{\gamma^2\lambda^2} = \frac{\rho}{3}$ , and the Hamiltonian equation gives the following identity:

$$\begin{aligned} \dot{V} = \{V, \mathcal{H}_{LQC}\} &= -\frac{\gamma}{2} \frac{\partial \mathcal{H}_{LQC}}{\partial \beta} \iff \\ H = \frac{\sin(2\lambda\beta)}{2\gamma\lambda} &\iff \beta = \frac{1}{2\lambda} \arcsin(2\lambda\gamma H). \end{aligned} \quad (16)$$

Writing this last equation as follows  $H^2 = \frac{\sin^2(\lambda\beta)}{\gamma^2\lambda^2}(1 - \sin^2(\lambda\beta))$  and using the Hamiltonian constraint  $\mathcal{H}_{LQC} = 0 \iff \frac{\sin^2(\lambda\beta)}{\gamma^2\lambda^2} = \frac{\rho}{3}$  one obtains the following modified Friedmann equation in loop quantum cosmology

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_c}\right) \iff \frac{H^2}{\rho_c/12} + \frac{(\rho - \frac{\rho_c}{2})^2}{\rho_c^2/4} = 1, \quad (17)$$

being  $\rho_c \equiv \frac{3}{\gamma^2\lambda^2}$ . This equation with the conservation equation  $\dot{\rho} = -3H(\rho + P)$  gives the dynamics of the universe in loop quantum cosmology.

Here two remark are in order:

1. The Hamiltonian (15) can be actually constructed by using the general formulae of loop gravity that express the Hamiltonian in terms of holonomies  $h_j(\lambda) \equiv e^{-i\frac{\lambda\beta}{2}\sigma_j}$  where  $\sigma_j$  are the Pauli matrices [21, 22, 23]:

$$\begin{aligned} \mathcal{H}_{LQG} \equiv & -\frac{2V}{\gamma^3\lambda^3} \sum_{i,j,k} \varepsilon^{ijk} \text{Tr} \left[ h_i(\lambda) h_j(\lambda) h_i^{-1}(\lambda) \right. \\ & \left. \times h_j^{-1}(\lambda) h_k(\lambda) \{h_k^{-1}(\lambda), V\} \right] + \rho V. \end{aligned} \quad (18)$$

A simple calculation shows, see for instance [24, 25], that (18) equals (15).

2. The old quantization of loop quantum cosmology was done using two canonically conjugate variables, one of them was the dynamical part of the connection, namely  $\mathfrak{c}$ , and the other one was the dynamical part of the triad, namely  $\mathfrak{p}$ , (see for instance [26, 27, 28]). These variables are related with the scalar factor and the extrinsic curvature  $K = \frac{1}{2}\dot{a}$  by the relations

$$\mathfrak{p} = a^2, \quad \mathfrak{c} = \gamma K = \frac{\gamma}{2}\dot{a}. \quad (19)$$

Then, in order to obtain the dynamics of the universe, the following effective Hamiltonian was used [29, 30, 31]

$$\mathcal{H}_{OLC} \equiv -\frac{3}{\gamma^2\mu^2} \mathfrak{p}^{1/2} \sin^2(2\mu\mathfrak{c}) + \rho \mathfrak{p}^{3/2}, \quad (20)$$

where  $\mu = \frac{3\sqrt{3}}{2}$  (see [23]) is obtained by identifying the eigenvalue  $\frac{\gamma\mu}{6}$  of the operator  $\hat{\mathfrak{p}}$  with the minimum eigenvalue of the area operator in loop quantum gravity which is given by  $\frac{\sqrt{3}}{4}\gamma$ .

Using this Hamiltonian, the scalar factor satisfies the dynamical equation

$$\dot{a} = \{a, \mathcal{H}_{OLC}\} = \frac{\sin(4\mu\mathfrak{c})}{2\mu\gamma}. \quad (21)$$

and, imposing once again the Hamiltonian constraint,  $\mathcal{H}_{OLC} = 0$ , one obtains the following modified Friedmann equation

$$H^2 = \frac{\rho}{3} \left( 1 - \frac{\rho(a)}{\rho_c(a)} \right), \quad (22)$$

with critical density

$$\rho_c(a) = \frac{3}{\gamma^2\mu^2 a^2}. \quad (23)$$

Coming back to the modified Friedmann equation in loop cosmology, (equation (17)), from the equation of the ellipsis one can see that the Hubble parameter belong in the interval  $[-\rho_c/12, \rho_c/12]$ , and the energy density  $\rho$  in  $[0, \rho_c]$ . Then, if the state equation  $P = f(\rho)$  is smooth enough, the functions  $H(t)$  and  $\rho(t)$  will be smooth and they also will be defined for all time, that is, there won't singularities.

As an example, we consider the solvable case  $P = \omega\rho$ . The dynamical equations are now

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_c}\right), \quad \dot{\rho} = -3H(1 + \omega)\rho, \quad (24)$$

which solution is given by

$$\rho(t) = \left(\frac{3}{4}(1 + \omega)^2(\bar{t} - t)^2 + \frac{1}{\rho_c}\right)^{-1}, \quad H(t) = \frac{1 + \omega}{2}(t - \bar{t})\rho(t) \quad (25)$$

where  $\bar{t} = t_0 - \frac{2\sqrt{1 - \frac{\rho_0}{\rho_c}}}{\sqrt{3(1 + \omega)}\sqrt{\rho_0}}$  and  $\rho_0$  is the current energy density of our universe. Note that this solution is defined for all time and it finishes at  $(H = 0, \rho = 0)$ , and its main property is that the universe remains in the expanding phase until time  $t = \bar{t}$ . At this time it bounces and re-collapses forever and ever.

It is clear that the behavior described by equation (25), is very different to the classical one (7), where the universe presents a big rip singularity.

From this result, it seems that holonomy corrections replace the big rip singularity, which appears in classical cosmology, by a non-singular bounce. However, from our viewpoint, we have some objections to the way that the modified Friedmann equation has been obtained.

1. Loop quantum cosmology was built using two canonically conjugate variables, one is the dynamical part of the connection, namely  $\mathfrak{c}$ , and the other one is the dynamical part of the triad, namely  $\mathfrak{p}$ , (see for instance [26, 27, 28]). These variables are related with the scalar factor and the extrinsic curvature  $K = \frac{1}{2}\dot{a}$  by the relations (see Section II of [27] and Section II of [28])

$$\mathfrak{p} = a^2, \quad \mathfrak{c} = \gamma K = \frac{\gamma}{2}\dot{a}. \quad (26)$$

Later in [32] two new canonically conjugate variables were introduced,  $(V, \beta)$  which are related with the standard variables through the relations  $V = a^3$  and  $\beta = \gamma H$  (see formulas 2.1 and 2.2 of [32]). Then, the loop quantum theory built with this two variables provides the effective Hamiltonian (15). However, if one starts directly from the effective Hamiltonian (15), although it is assumed that  $V = a^3$ , the definition of the variable  $\beta$  comes from the equation  $\dot{V} = \{V, \mathcal{H}_{LQC}\}$ , which gives  $\beta = \frac{1}{2\lambda} \arcsin(2\lambda\gamma H)$  and differs from the initial definition of  $\beta$ . As a consequence, if one takes  $V = a^3$  and

$\beta = \frac{1}{2\lambda} \arcsin(2\lambda\gamma H)$  as a canonically conjugate variables, then the standard variables  $(a, p = -6Ha^2)$  do not remain canonically conjugate, because now

$$\{a, p\} \equiv \frac{\gamma}{2} \left( \frac{\partial a}{\partial \beta} \frac{\partial p}{\partial V} - \frac{\partial a}{\partial V} \frac{\partial p}{\partial \beta} \right) = \cos(2\lambda\beta) = \sqrt{1 - 4\gamma^2\lambda^2 H^2} \neq \text{constant}. \quad (27)$$

2. One of the main reason against the modified Friedmann equation in loop quantum cosmology comes from the Legendre transformation

$$\mathcal{H}_{LQC} = -\frac{2}{\gamma} \dot{V} \beta - \mathcal{L}_{LQC} \quad (28)$$

which gives, in terms of the standard variables, the following Lagrangian

$$\mathcal{L}_{LQC} = -\frac{3a^3 H}{\gamma\lambda} \arcsin(2\lambda\gamma H) + \frac{3a^3}{2\gamma^2\lambda^2} \left( 1 - \sqrt{1 - 4\gamma^2\lambda^2 H^2} \right) - a^3 \rho, \quad (29)$$

which coincides with  $\mathcal{L}_E$  for small values of  $H$ .

It's well-known that the other current cosmological theories are built from two invariant, the scalar curvature  $R = 6(\dot{H} + 2H^2)$  and the Gauss-Bonnet curvature invariant  $G = 24H^2(\dot{H} + H^2)$ . For example, in modified Gauss-Bonnet gravity [13] the Lagrangian  $\mathcal{L}_{GB} = a^3 f(R, G) - a^3 \rho$  is used, and semiclassical gravity, when one takes into account the quantum effects due to a massless conformally coupled field (see for instance [14]), is based in the trace anomaly  $T_{vac} = \alpha \square R - \frac{\beta}{2} G$  (being  $\alpha > 0$  and  $\beta < 0$  two renormalization coefficients). However, the Lagrangian (29) does not seem invariant, which is in disagreement with one of the main principles of general relativity.

3. In semiclassical gravity or in the  $f(R, G) = \frac{R}{2} - \frac{\alpha}{12} R^2$  theory, the modified Friedmann equation is given by

$$H^2 = \frac{\rho}{3} + 2\alpha(3H^2\dot{H} + H\ddot{H} - \frac{1}{2}\dot{H}^2) - \beta H^4, \quad (\beta = 0 \text{ in the } f(R, G) = \frac{R}{2} - \frac{\alpha}{12} R^2 \text{ theory}), \quad (30)$$

which contains higher-curvature terms like  $\dot{H}$  and  $\ddot{H}$ . Using this equation, it was proved in [14] for the state equation  $P = \omega\rho$  that, if  $-1 \leq \frac{\beta}{3\alpha} \leq 0$  the universe will bounce but when it will enter in the contracting phase it will develop a future singularity of the form

$$H(t) \sim \frac{3\alpha}{\beta} \left( -1 \pm \sqrt{1 + \frac{\beta}{3\alpha}} \right) \frac{1}{(t_s - t)}, \quad \rho(t) \sim 0, \quad \text{for } t < t_s \quad (31)$$



being  $t_s$  the time at which the future singularity appears. It is clear that this behavior is very different to the one described in loop quantum cosmology (eq. (25)), where the universe does not develop any kind of singularity.

On the other hand, one can adopt another different point of view. One can assume that the variables  $(a, p = -6Ha^2)$  are canonically conjugate, which means that  $V = a^3$  and  $\beta = \gamma H$  as we have seen at the beginning of this Section. Then, one has to understand the Hamiltonian (15), not like the Hamiltonian of the system, but as the new Hamiltonian constraint that replaces the classical one. From this viewpoint, taking the derivative with respect to the time of the Hamiltonian constraint and finally using the conservation equation, the following modified Raychaudury equation will be obtained

$$\dot{H} = -\frac{\lambda\gamma H}{\sin(2\lambda\gamma H)} \left[ \frac{3\sin^2(\lambda\gamma H)}{\lambda^2\gamma^2} + f\left(\frac{3\sin^2(\lambda\gamma H)}{\lambda^2\gamma^2}\right) \right]. \quad (32)$$

For the case  $P = f(\rho) = \omega\rho$  this equation becomes

$$\dot{H} = -\frac{3}{2}(1+\omega)H \frac{\tan(\gamma\lambda H)}{\gamma\lambda}, \quad (33)$$

from which one deduces that  $\dot{H}$  is positive, which means that  $H$  reach the value  $\frac{\pi}{2\gamma\lambda}$  in a finite time and thus, at that time  $\dot{H}$  diverges or equivalently the scalar curvature  $R = 6(\dot{H} + 2H^2)$  diverges. Moreover, at that time, from the Hamiltonian constraint and the state equation  $P = \omega\rho$ , one has  $\rho = \frac{3}{\gamma^2\lambda^2}$  and  $P = \frac{3\omega}{\gamma^2\lambda^2}$ . Then, one can conclude that, from this viewpoint, the big rip singularity is replaced by this other singularity characterized by a divergent scalar curvature, but with finite values of the Hubble parameter, energy density and pressure.

*5.- Theory based on the reduced semiclassical Friedmann equation—* This approach was proposed by Parker and Simon in [33, 34], its main idea is to obtain the derivatives of  $H$  from the classical Friedmann and conservation equations. Thus, once this has been done, one inserts these derivatives into the semiclassical Friedmann equation obtained in modified gravity (equation (10)). The result is a new modified Friedmann equation, but without derivatives on  $H$ . To be precise, we consider the theory  $f(R) = \frac{R}{2} - \frac{\alpha}{12}R^2$ . Then, for the simplest case  $P = \omega\rho$  we have  $\dot{H} = -\frac{3}{2}(1+\omega)H^2$  and  $\ddot{H} = \frac{9}{2}(1+\omega)^2H^3$ , that once introduced in (11) provides

$$H^2 = \frac{\rho}{3} + \frac{9}{4}\alpha(1+\omega)(3\omega-1)H^4. \quad (34)$$

If in this equation one makes the substitution  $H^4 = \frac{\rho^2}{9}$  (classical Friedmann equation) one will get the interesting equation

$$H^2 = \frac{\rho}{3} + \frac{1}{4}\alpha(1+\omega)(3\omega-1)\rho^2, \quad (35)$$

which gives a way to obtain the modified Friedmann equations in loop quantum cosmology. Effectively, choosing  $\alpha = -\frac{4\rho_c(1+\omega)(3\omega-1)}{3}$  one obtains the equation (17). Then, in some sense, for the particular state equation  $P = \omega\rho$ , this approach could justify the modified Friedmann equation in loop quantum cosmology. However, note that, for a more general state equation  $P = f(\rho)$ , this method provides more complicated equations than (35), and thus, it is impossible to recover equation (17).

**6.- Conclusions**— Through this paper we have shown that it seems impossible to avoid completely the future singularities, which appear in phantom dark energy models, using different alternative approaches to quantum cosmology. We think the most efficient approach is semiclassical or equivalently modified gravity with  $f(R) = \frac{R}{2} - \frac{\alpha}{12}R^2$  (with  $\alpha > 0$ ). In this theory the dynamics of the universe is drastically changed because the universe will bounce and will enter in the contracting phase where it will develop a future singularity like that described by equation (12) or (31). The other important conclusion in the paper, is that the results about avoidance of future singularities, in particular the bounces, obtained from the modified Friedmann equation in loop quantum cosmology (for example [35, 36, 37]) have to be revisited because this equation would not be justified. This does not mean that, in a more general theory where higher-curvature terms would be combined with holonomy corrections, features such as bounces may appear. But this is a complicated problem that deserves future investigations.

**Acknowledgments.** I would like to thank Prof. Martin Bojowald for your critical comments and suggestions about the first version of this paper which have been very important in order to improve it. This investigation has been supported in part by MICINN (Spain), project MTM2011-27739-C04-01, and by AGAUR (Generalitat de Catalunya), contract 2009SGR-345.

## References

- [1] S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999).
- [2] A.G. Riess et al., *Astron. J.* **116**, 1009 (1999).
- [3] E. Kamatsu et al., *Astrophys. J. Suppl. Ser.* **192**, 18 (2011).
- [4] R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, *Phys. Rev. Lett.* **91**, 071301 (2003).
- [5] J.D. Barrow, *Class. Quantum. Grav.* **21**, L79 (2004); J.D. Barrow, *Class. Quantum Grav.* **21**, 5619 (2004).
- [6] S. Nojiri, S. Odintsov and S. Tsujikawa, *Phys. Rev.* **D71**, 063005 (2005).

- [7] J. Socorro, IJTP **42**, 2087 (2003).
- [8] M. Ryan Jr., *Hamiltonian Cosmology*, Springer-Verlag (1972).
- [9] L. Amendola, D. Polarski and S. Tsujikawa, Phys. Rev. Lett **98**, 131302 (2007).
- [10] S. Nojiri and S. Odintsov, Phys. Rev. **D68**, 123512 (2003).
- [11] K. Bamba, S. Nojiri and S. Odintsov, JCAP **10**, 045 (2008).
- [12] S. I. Muslih and H. A. El-Zalan, Int. Jour. Theor. Phys. **46**, 3150 (2007).
- [13] G. Cognola, E. Elizalde, S. Nojiri, S. Odintsov and S. Zerbini, Phys. Rev. **D73**, 084007 (2006).
- [14] J. Haro, J. Amoros and E. Elizalde, Phys. Rev. **D83**, 123528 (2011).
- [15] A. Ashtekar and P. Singh, Class. Quantum Grav. **29**, 213001 (2011).
- [16] P. Singh, Class. Quantum Grav. **26**, 125005 (2009).
- [17] P. Singh, J. Phys. Conf. Ser. **140**, 012005 (2009).
- [18] K.A. Mieissner, Class. Quantum Grav. **21**, 5245 (2004).
- [19] A. Corichi and P. Singh, Phys. Rev. **D80**, 044024 (2009).
- [20] P. Malkiewicz and W. Piechocki, *Observables for FRW model with cosmological constant in the framework of loop cosmology*, gr-qc/1001.3999v3 (2010).
- [21] A. Ashtekar, M. Bojowald and J. Lewandowski, Adv. Theor. Math. **7**, 233 (2003).
- [22] T. Thiemann, *Introduction to modern canonical quantum general relativity*, gr-qc/0110034 (2001).
- [23] A. Ashtekar, T. Pawłowski and P. Singh, Phys. Rev. **D73**, 124038 (2006).
- [24] J. Haro and E. Elizalde, EPL **89**, 69001 (2010).
- [25] P. Dzierzak, P. Malkiewicz and W. Piechocki, Phys. Rev. **D80**, 104001 (2009).
- [26] M. Bojowald, Phys. Rev. Lett. **89**, 261301 (2002).
- [27] M. Bojowald and K. Vanderslot, Phys. Rev. **D67**, 124023 (2003).
- [28] F. Cianfrani and G. Montani, Phys. Rev. **D82**, 021501(R) (2010).
- [29] P. Singh, Phys. Rev. **D73**, 063508 (2006).

- [30] M. Bojowald, Class. Quantum Grav. **26**, 075020 (2009).
- [31] P. Singh and K. Vandersloot, Phys. Rev. **D72**, 084004 (2005).
- [32] A. Ashtekar, C. Corichi and P. Singh, Phys. Rev. **D77**, 024046 (2008).
- [33] J.Z. Simon, Phys. Rev. **D45**, 1953 (1992); **D41**, 3720 (1990).
- [34] L. Parker and J.Z. Simon, Phys. Rev. **D47**, 1339 (1993).
- [35] M. Sami, P. Singh and S. Tsujikawa, Phys. Rev. **D74**, 043514 (2006).
- [36] T. Naskar and J. Ward, Phys. Rev. **D76**, 063514 (2007).
- [37] D. Smart and B. Gumjudpai, Phys. Rev. **D76**, 043514 (2007).